# SPOC: An Innovative Beamforming Method

***Benjamin Shapo***
General Dynamics—Advanced Information Systems
phone: 734-994-1200
email: ben.shapo@gd-ais.com

***Roy Bethel***
MITRE
email: rbethel@mitre.org

**Abstract** We recently introduced SPOC (Spatial Processing: Optimized and Constrained) [1] as an innovative replacement for the spatial spectrum estimation traditionally done by beamformers. This work extends the method described in [1] by improving the formalism and presenting results with at-sea sonar data.

Beamforming has traditionally been performed by conventional (CBF) or adaptive (ABF) processing. These both provide estimates of spatial spectrum in a given time interval (snapshot). A trade-off between detection and resolution governs CBF performance. ABF offers improvement by lessening the effects of interfering signals at each beam. This is done with an estimated covariance matrix which involves a trade-off in selecting the averaging interval. Long averaging times produce better estimates of the true covariance, but shorter intervals decrease the harmful impact of target dynamics.

SPOC produces spatial spectrum estimates without using covariances. It considers an array of $N$ sensors and a finite but large number of sources of energy $L$ ($L >> N$) spanning the entire Direction of Arrival (DOA) space ($-1 \leq \cos\theta \leq 1$). SPOC is a constrained optimization approach to estimating the signals at each of the $L$ DOAs (or "beams"). The complex-valued signals are constrained so that the superposition of the weighted steering vectors at each DOA plus the estimated noise equals the measured data. Many sets of signals generally satisfy the constraint so the minimum-energy set of signals is chosen. Mathematical details will appear in the full-length paper.

As a preliminary demonstration of performance improvement over ABF, we compared APB-98 passive sonar data (system ABF) versus the same data processed using SPOC. The processing was done on every frequency bin at the system resolution and broadband processing done on both using the same standard "SPED CS" algorithm [2]. The passive broadband grams demonstrate that more contacts are visible with SPOC processing than with ABF.

[1] R. Bethel, B. Shapo, and H. Van Trees, "Single Snapshot Spatial Processing: Optimized and Constrained," Sensor Array and Multichannel Signal Processing Workshop 2002 (SAM 2002) Proceedings, pp. 508–512.

[2] M. Bono, R. Bethel, P. McCarty, and B. Shapo, "Subband Energy Detection Methods in Passive Array Processing," MIT Lincoln Laboratory ASAP Workshop Proceedings, 2001.

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# SPOC: An Innovative Beamforming Method

*Benjamin Shapo*
General Dynamics
Ann Arbor, MI
ben.shapo@gd-ais.com

*Roy Bethel*
The MITRE Corporation
McLean, VA
rbethel@mitre.org

## ABSTRACT

***The purpose of a radar or sonar system is to detect targets of interest. Beamforming plays a pivotal role in this task. Spatial Processing: Optimized and Constrained (SPOC) is an innovative front-end beamforming method that operates on array sensor data as an alternative to conventional or adaptive beamforming. It uses a maximum a posteriori approach to develop a constrained optimization solution. A parametric representation provides the framework for the solution presented here. We develop a recursive solution using past history. This method provides significantly improved resolution over conventional and adaptive beamforming. Three real data sets demonstrate these performance improvements.***

## 1. INTRODUCTION

We recently introduced SPOC ("Spatial Processing: Optimized and Constrained") [1] as an innovative replacement for the spatial spectrum estimation traditionally done by beamformers. This work extends the method described in [1] by improving the formalism and presenting results with at-sea sonar data.

Beamforming has traditionally been performed by conventional (CBF) or adaptive (ABF) processing. These both provide estimates of spatial spectrum in a given time interval (snapshot). A tradeoff between detection and resolution governs CBF performance. ABF offers improvement by lessening the effects of interfering signals at each beam. This is done with an estimated covariance matrix, which involves a tradeoff in selecting the averaging interval. Long averaging times produce better estimates of the true covariance but shorter intervals decrease the harmful impact of target dynamics.

SPOC produces spatial spectrum estimates without using covariances. It considers an array of $N$ sensors and a finite but large number of sources of energy $L$ ($L$>>$N$) spanning the entire Direction of Arrival (DOA) space ($-1 \le \cos(\theta) \le 1$). SPOC is a constrained optimization approach to estimating the signals at each of the $L$ DOAs (or "beams"). The complex-valued signals are constrained so that the superposition of the weighted steering vectors at each DOA plus the estimated noise equals the measured data. Many sets of signals generally satisfy the constraint so the minimum energy set of signals is chosen.

As a demonstration of performance improvement over ABF and CBF, we compared passive sonar data versus the same data processed using SPOC. The processing was done on every frequency bin and broadband processing done on both using the same standard algorithm [2]. The passive broadband displays in the results section demonstrate that more contacts are visible with SPOC processing than with ABF or CBF.

## 2. APPROACH

We consider the sensor array problem with complex narrowband data. We assume that the spatial energy impinging upon the sensor array emanates from a dense grid of far field independent sources, each radiating a narrowband signal. In the limit, the set of discrete sources approaches a continuum. This is a common representation of spatial energy [2]. The goal is to estimate the energy of each discrete source.

For implementation purposes, we consider a uniformly spaced linear array of $N$ sensors. We assume a finite but large number of sources of energy $L$ ($L \gg N$) uniformly spanning the entire Direction of Arrival (DOA) space $u$ $(-1 \le u \le +1)$. Each sensor $n$ ($n = 1,\ldots,N$) receives the sum of all narrowband signals radiated by sources $l$ at DOA's $u_l$ ($l = 1,\ldots,L$) for all $L$ sources. This observation model is expressed as:

$$\underline{x} = \sum_{l=1}^{L} \underline{v}_l(u_l) f_l = \underline{\underline{V}}(\underline{u}) \underline{f}, \tag{1}$$

where $\underline{x}$ is the $N$x1 complex narrowband sensor data, $\underline{v}(u_l)$ is the $N$x1 array manifold vector for DOA $u_l$, $\underline{\underline{V}}(\underline{u})$ is the $N$x$L$ array manifold matrix, $f_l$ is the scalar complex-valued $l^{th}$ signal, and $\underline{f}$ is the $L$x1 signal vector.
$|f_l|^2$ is the signal power at the $l^{th}$ discrete DOA. To accomplish our goal, we need an estimate of these

quantities. This first requires estimates of $f_l$. With no knowledge of observed sensor data $\underline{x}$, the signal vector $\underline{f}$ is expressed in the *a priori* Probability Density Function (PDF) p( $\underline{f}$ ), which is assumed zero-mean complex Gaussian:

$$\mathrm{p}(\underline{f}) = \frac{1}{\pi^L \left| \underline{\underline{C}}_f \right|} \exp\left\{ -\underline{f}^{\mathrm{H}} \underline{\underline{C}}_f^{-1} \underline{f} \right\}, \qquad (2)$$

where $(\bullet)^{\mathrm{H}}$ represents the conjugate transpose operation, and $\underline{\underline{C}}_f$ is the $L$x$L$ signal covariance matrix E[ $\underline{f}\underline{f}^{\mathrm{H}}$ ]. Under the assumption of independent signals, $\underline{\underline{C}}_f = \mathrm{diag}(\sigma_l^2)$, where $\sigma_l^2$ is the expected power of the $l^{\text{th}}$ signal, which is unknown.

We use observed sensor data $\underline{x}$ to improve knowledge of signal vector $\underline{f}$ via the *a posteriori* PDF p( $\underline{f} \mid \underline{x}$ ). In particular, we find the maximum *a posteriori* (MAP) estimate of signal vector $\underline{f}$ by maximizing p( $\underline{f} \mid \underline{x}$ ) with respect to $\underline{f}$ with $\underline{x}$ known. The MAP signal power estimate of each $|f_l|^2$ is computed from the MAP estimate of $f_l$. This is accomplished by Bayes' theorem:

$$\mathrm{p}(\underline{f} \mid \underline{x}) = A\, \mathrm{p}(\underline{x} \mid \underline{f})\, \mathrm{p}(\underline{f}). \qquad (3)$$

Scalar $A$ is a normalizing constant and does not affect the computation of MAP estimates. The conditional PDF p($\underline{x} \mid \underline{f}$) is required in (3). We note that (1) must be satisfied. Therefore, we must restrict signal vector $\underline{f}$ to values which meet this requirement. The implication in (3) is that the conditional PDF p( $\underline{x} \mid \underline{f}$ ) is zero for any signal vector $\underline{f}$ which does not satisfy the observation model (1), and conditional PDF p( $\underline{x} \mid \underline{f}$ ) is equally likely for any signal vector $\underline{f}$ which does satisfy (1).

Therefore, to find the MAP estimate of signal vector $\underline{f}$, we must maximize the *a priori* PDF p($\underline{f}$) in (2) subject to the constraint that (1) is satisfied.

Substituting this result into (4) yields:

$$\text{Cost Function}: \quad \Gamma = \underline{f}^H \underline{\underline{C}}_f^{-1} \underline{f} \qquad (6a)$$

$$\text{Constraint}: \quad \underline{x} = \underline{\underline{V}}\,\underline{f}, \qquad (6b)$$

where $\underline{\underline{C}}_f$ is the theoretical diagonal covariance matrix for the signal vector $\underline{f}$, and the constant term in (4) has been neglected. Also, the expression (4) has been negated, resulting in a cost-minimization problem.

Then, the complex signal vector $\underline{f}$ which minimizes the cost $\Gamma$ in (6a), subject to the constraint in (6b), is the MAP estimate. Signal powers $|f_l|^2$ for all DOA's $u_l$ are the desired output and are the MAP estimates for signal powers.

The spatial spectrum is defined as the Fourier transform of the stationary spatial autocorrelation function. We emphasize that SPOC is not a spatial spectrum estimator in this sense. SPOC also differs greatly from CBF and ABF in that it is not a spatial filter. It does not require an estimated covariance matrix as does ABF. Also, SPOC is neither a source detection technique [4], nor a DOA estimation algorithm [5, 6, 7]. The results in Section 4 demonstrate the performance characteristics of SPOC.

## 3. IMPLEMENTATION

The desired quantity in (6b) is the vector of signals $\underline{f}$. The solution to (6a) and (6b) is:

$$\underline{\hat{f}} = \underline{\underline{C}}_f \underline{\underline{V}}^H (\underline{\underline{V}}\,\underline{\underline{C}}_f \underline{\underline{V}}^H + \sigma^2 \underline{\underline{I}})^{-1} \underline{x}, \qquad (7)$$

where $\sigma^2$ is diagonal loading for numerical stability. The desired output is the signal vector power $\left|\underline{\hat{f}}\right|^2$, on a per-scan basis. $\underline{\underline{C}}_f$. An estimate of the diagonal matrix $\underline{\underline{C}}_f$ is required in (7). This is obtained by the computing the diagonal of $\underline{\underline{C}}_f$ as an exponentially averaged version of $\left|\underline{\hat{f}}\right|^2$.

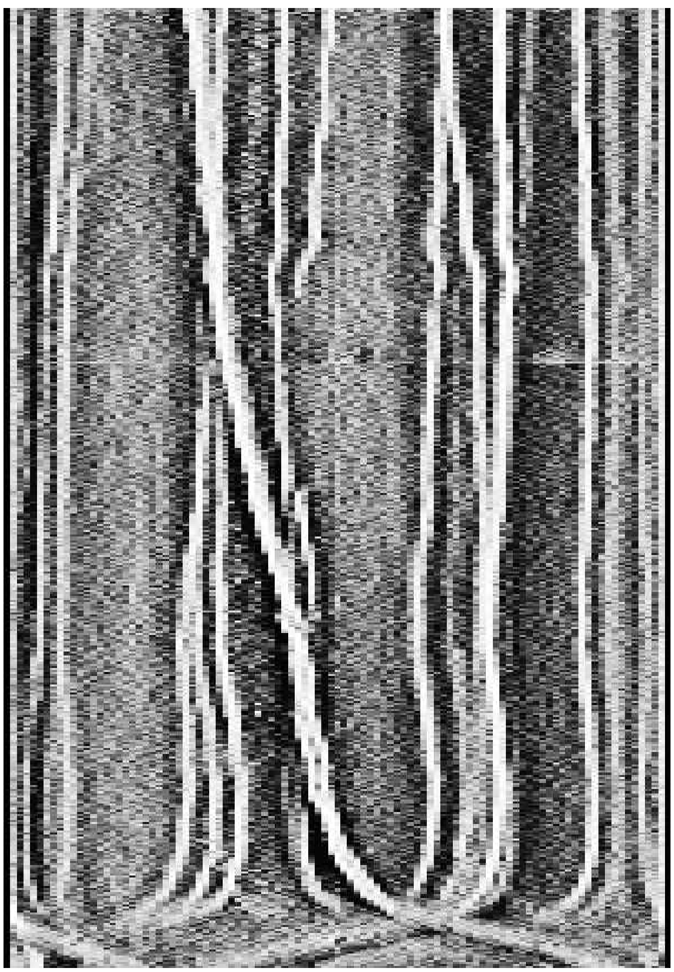
To take dynamic conditions into account, a smoothing operation can be applied over DOA. In practice, we have applied a simple FIR filter over beams to the signal vector power $\left|\underline{\hat{f}}\right|^2$. This is roughly analogous to the assumption that the changes in the signal DOA values follow a first-order Markov process.

## 4. RESULTS

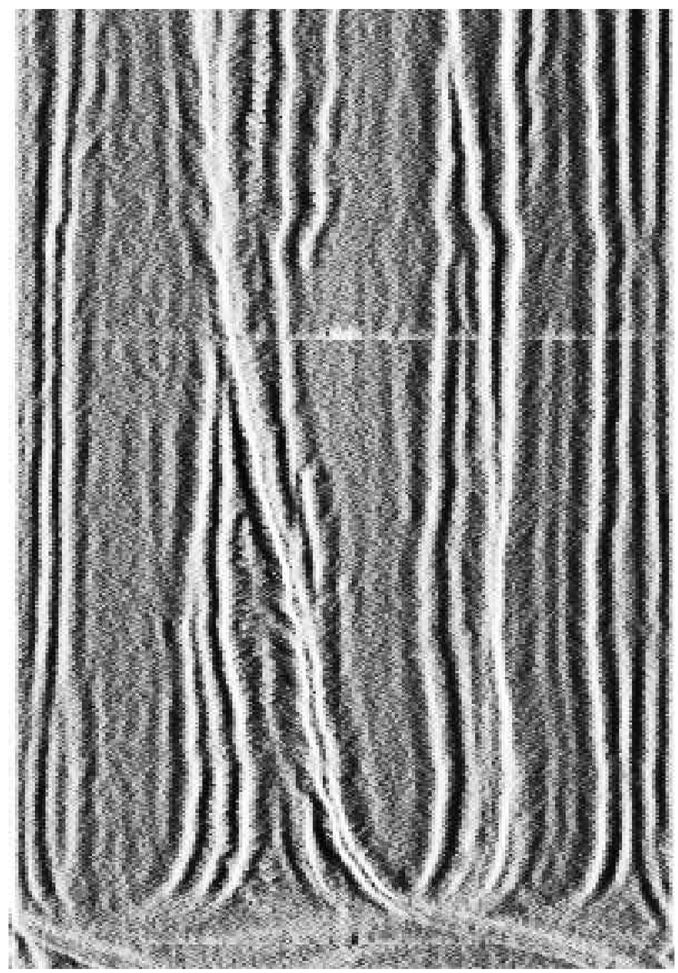
We implemented this algorithm on developmental testbed with real passive acoustic data. The results are fair comparisons between SPOC and CBF or SPOC and a beamspace MVDR ABF. The single-frequency SPOC processing described above is implemented independently for each frequency bin over a broadband bandwidth. A standard broadband processing technique results in the bearing versus time data displays below. Energy is intensity modulated where white indicates stronger energy. The horizontal axis corresponds to bearing (DOA) and the vertical axis represents time. Three data sets are presented.

These figures all demonstrate that contacts missing in both ABF and CBF are detected in SPOC. This is a significant improvement in beamforming performance.

# DATA SET 1



ABF



SPOC

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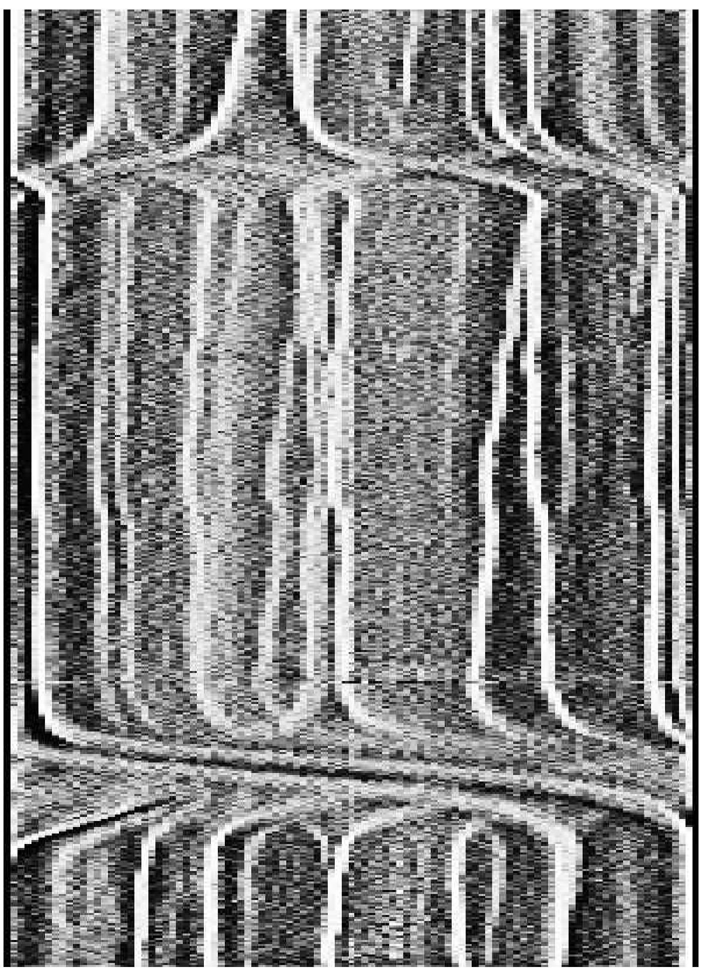
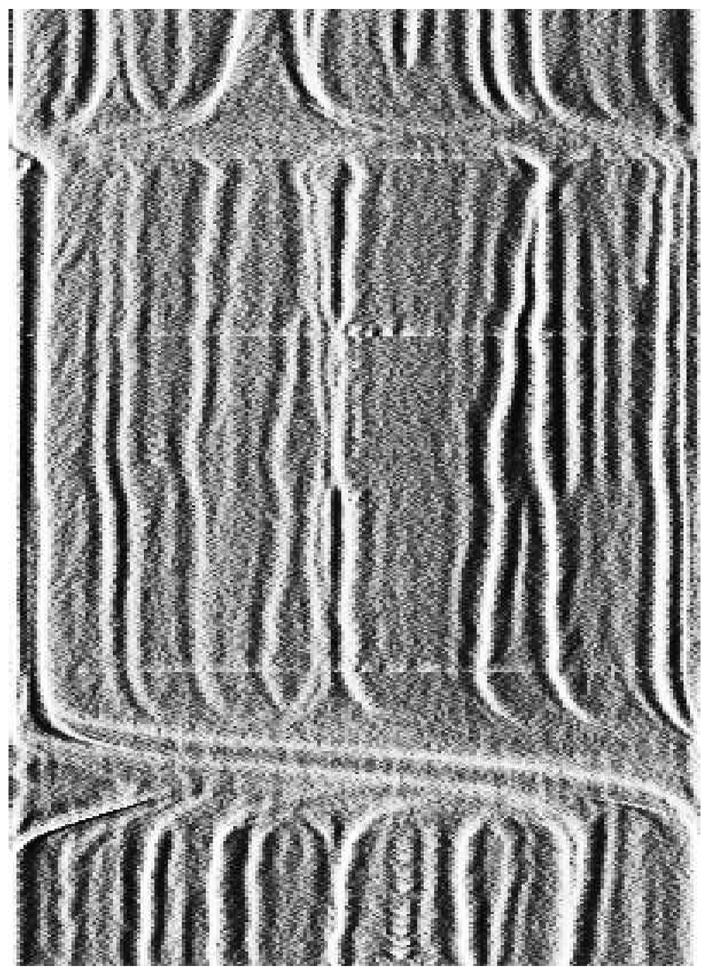


ABF



SPOC

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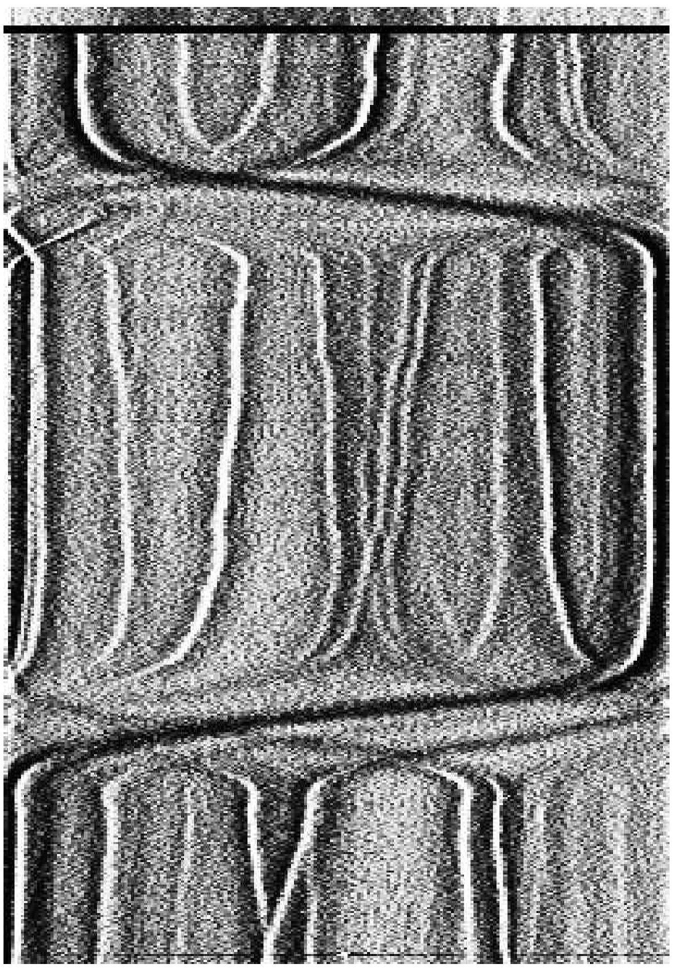
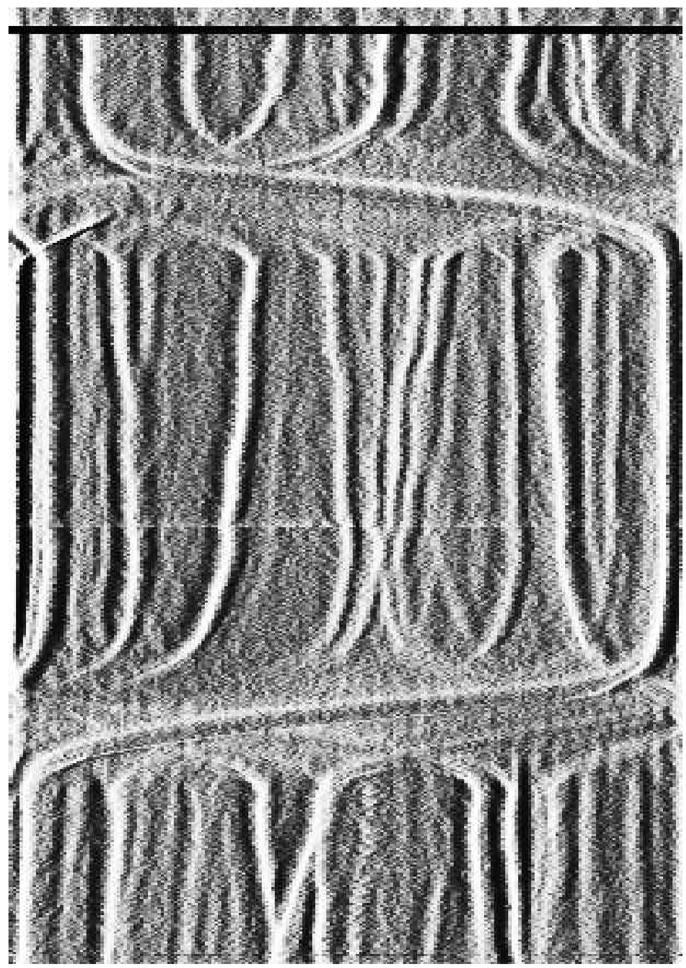


CBF



SPOC

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